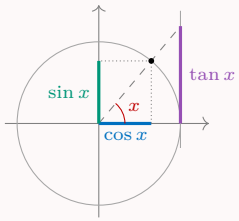
FORMULAIRE · TRIGONOMÉTRIE

Dans toute la page, a, b, p, q, x sont des réels (expressions supposées définies) et $n \in \mathbb{N}$. **MPSI · MP**

CERCLE



VALEURS REMARQUABLES

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\tan x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	—

SYMÉTRIES

y	$-x$	$\pi - x$	$\pi + x$	$\frac{\pi}{2} - x$	$\frac{\pi}{2} + x$
$\cos y$	$\cos x$	$-\cos x$	$-\cos x$	$\sin x$	$-\sin x$
$\sin y$	$-\sin x$	$\sin x$	$-\sin x$	$\cos x$	$\cos x$
$\tan y$	$-\tan x$	$-\tan x$	$\tan x$	$\frac{1}{\tan x}$	$\frac{-1}{\tan x}$

RELATIONS FONDAMENTALES, DÉRIVÉES ET PRIMITIVES

$\cos$ → paire, 2π-périodique — $\sin$ → impaire, 2π-périodique — $\tan$ → impaire, π-périodique

$$\cos^2 x + \sin^2 x = 1 \qquad 1 + \tan^2 x = \frac{1}{\cos^2 x}$$

$$\cos' = -\sin \qquad \sin' = \cos$$

$$\tan' = 1 + \tan^2 = \frac{1}{\cos^2} \qquad \operatorname{Arcsin}' x = \frac{1}{\sqrt{1-x^2}} \qquad \operatorname{Arccos}' x = \frac{-1}{\sqrt{1-x^2}}$$

$$\operatorname{Arctan}' x = \frac{1}{1+x^2} \qquad \operatorname{Arcsin} x + \operatorname{Arccos} x = \frac{\pi}{2} \qquad \operatorname{Arctan} x + \operatorname{Arctan} \frac{1}{x} = \pm\frac{\pi}{2} \text{ si } \pm x > 0$$

$$\int \tan x \,\mathrm{d}x = -\ln|\cos x| \qquad \int \frac{\mathrm{d}x}{\sin x} = \ln\left|\tan\frac{x}{2}\right| \qquad \int \frac{\mathrm{d}x}{\cos x} = \ln\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right|$$

ADDITION, ANGLE DOUBLE ET ANGLE MOITIÉ

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b \qquad \sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b} \qquad \sin(2x) = 2\sin x \cos x \qquad \tan(2x) = \frac{2\tan x}{1 - \tan^2 x}$$

$$\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x \qquad \cos^2 x = \frac{1 + \cos(2x)}{2} \qquad \sin^2 x = \frac{1 - \cos(2x)}{2}$$

$$\text{Avec } t = \tan\frac{x}{2} : \quad \cos x = \frac{1-t^2}{1+t^2}, \quad \sin x = \frac{2t}{1+t^2}, \quad \tan x = \frac{2t}{1-t^2}.$$

PRODUITS, SOMMES ET DIFFÉRENCES

$$\cos a \cos b = \frac{1}{2}\big(\cos(a-b) + \cos(a+b)\big) \qquad \cos p + \cos q = 2\cos\frac{p+q}{2}\cos\frac{p-q}{2}$$

$$\sin a \sin b = \frac{1}{2}\big(\cos(a-b) - \cos(a+b)\big) \qquad \cos p - \cos q = -2\sin\frac{p+q}{2}\sin\frac{p-q}{2}$$

$$\sin a \cos b = \frac{1}{2}\big(\sin(a+b) + \sin(a-b)\big) \qquad \sin p \pm \sin q = 2\sin\frac{p \pm q}{2}\cos\frac{p \mp q}{2}$$

$$a\cos x + b\sin x = r\cos(x - \varphi) \quad \text{où} \quad a + \mathrm{i}b = r\,\mathrm{e}^{\mathrm{i}\varphi} \quad (r \geqslant 0)\,; \quad \text{par exemple } \cos x + \sin x = \sqrt{2}\cos\left(x - \frac{\pi}{4}\right).$$

EXPONENTIELLE COMPLEXE, ÉQUATIONS ET SOMMES

$$\mathrm{e}^{\mathrm{i}x} = \cos x + \mathrm{i}\sin x \qquad \cos x = \frac{\mathrm{e}^{\mathrm{i}x} + \mathrm{e}^{-\mathrm{i}x}}{2} \qquad \sin x = \frac{\mathrm{e}^{\mathrm{i}x} - \mathrm{e}^{-\mathrm{i}x}}{2\mathrm{i}}$$

$$(\cos x + \mathrm{i}\sin x)^n = \cos(nx) + \mathrm{i}\sin(nx) \qquad \mathrm{e}^{\mathrm{i}p} + \mathrm{e}^{\mathrm{i}q} = 2\cos\frac{p-q}{2}\,\mathrm{e}^{\mathrm{i}\frac{p+q}{2}} \quad \text{(angle moitié)}$$

$$\mathrm{e}^{\mathrm{i}p} - \mathrm{e}^{\mathrm{i}q} = 2\mathrm{i}\sin\frac{p-q}{2}\,\mathrm{e}^{\mathrm{i}\frac{p+q}{2}} \qquad \sum_{k=0}^{n}\cos(kx) = \frac{\cos\frac{nx}{2}\,\sin\frac{(n+1)x}{2}}{\sin\frac{x}{2}} \quad (x \not\equiv 0\ [2\pi])$$

$$\cos x = \cos a \iff x \equiv \pm a\ [2\pi] \qquad \sin x = \sin a \iff x \equiv a \text{ ou } x \equiv \pi - a\ [2\pi] \qquad \tan x = \tan a \iff x \equiv a\ [\pi]$$

INÉGALITÉS ET DÉVELOPPEMENTS LIMITÉS EN 0

$$|\sin x| \leqslant |x| \qquad 1 - \frac{x^2}{2} \leqslant \cos x \leqslant 1$$

$$\frac{2}{\pi}x \leqslant \sin x \leqslant x \leqslant \tan x \quad \text{sur } \left[0, \frac{\pi}{2}\right[$$

$$\cos x = 1 - \frac{x^2}{2} + \cdots + (-1)^n\frac{x^{2n}}{(2n)!} + o\big(x^{2n+1}\big) \qquad \sin x = x - \frac{x^3}{6} + \cdots + (-1)^n\frac{x^{2n+1}}{(2n+1)!} + o\big(x^{2n+2}\big)$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + o\big(x^8\big) \qquad \operatorname{Arctan} x = x - \frac{x^3}{3} + \cdots + (-1)^n\frac{x^{2n+1}}{2n+1} + o\big(x^{2n+2}\big)$$